

ENGEL-TYPE CONDITIONS ON FIXED POINTS OF AUTOMORPHISMS OF FINITE AND PROFINITE GROUPS

EVGENY KHUKHRO

JOINT WORK WITH PAVEL SHUMYATSKY

ABSTRACT. A left Engel sink of an element g of a group G is a set $\mathcal{E}(g)$ such that for every $x \in G$ all sufficiently long commutators $[\dots[x, g], g], \dots, g]$ belong to $\mathcal{E}(g)$. A right Engel sink of an element g of a group G is a set $\mathcal{R}(g)$ such that for every $x \in G$ all sufficiently long commutators $[\dots[[g, x], x], \dots, x]$ belong to $\mathcal{R}(g)$. Thus, g is a left (right) Engel element precisely when we can choose $\mathcal{E}(g) = \{1\}$ (respectively, $\mathcal{R}(g) = \{1\}$). Earlier we proved that if all elements of a profinite (or compact) group have finite or countable left (or right) Engel sinks, then the group is finite-by-(locally nilpotent). Similar quantitative results hold for finite groups, where the order of a normal subgroup with nilpotent quotient is bounded in terms of m , given that every element has a left (right) Engel sink of cardinality at most m .

We prove similar structure theorems for finite and profinite groups admitting coprime groups of automorphisms whose non-trivial elements fix only elements with finite or countable left (or right) Engel sinks. For finite groups, the sizes of these Engel sinks are assumed to be at most m , and the conclusions involve bounds in terms of m .